**Lab Exercise 4 | Week 5**

Implement ***Gale–Shapley algorithm*** *to solve stable marriage problem*.

The ***stable marriage problem (SMP)*** has been stated as follows:

Given ***n*** men and ***n*** women, where each person has ranked all members of the opposite sex in order of preference, [marry](https://en.wikipedia.org/wiki/Marriage) the men and women together such that there are no two people of opposite sex who would both rather have each other than their current partners. When there are no such pairs of people, the set of marriages is deemed stable.

***Input*:** 2D matrix of size (2\*N)\*N where N is number of women or men. Rows from 0 to N-1 represent preference lists of men and rows from N to 2\*N – 1 represent preference lists of women. Men are numbered from 0 to N-1 and women are numbered from N to 2\*N – 1.

***Output***: The output is list of married pairs.

While the solution is stable, comment on the optimality of the solution. {Refer Wikipedia}

[Other variations in the SMP you can try out in the lab after implementing basic SMP.]

The **Stable Matching Problem**, as discussed in the class, assumes that all men and women have a fully ordered list of preferences. In this problem we will consider a version of the problem in which men and women can be indifferent between certain options. As before we have a set ***M*** of ***n*** men and a set ***W*** of ***n*** women. Assume each man and each woman ranks the members of the opposite gender, but now we allow ties in the ranking. For example (with ***n*** = 4), a woman could say that ***m1*** is ranked in first place; second place is a tie between ***m2*** and ***m3*** (she has no preference between them); and ***m4*** is in last place. We will say that ***w*** prefers ***m*** to ***m’*** if ***m*** is ranked higher than ***m’*** on her preference list (they are not tied). With indifferences in the rankings, there could be two natural notions for stability. And for each, we can ask about the existence of stable matchings, as follows.

1. A **strong instability** in a perfect matching ***S*** consists of a man ***m*** and a woman ***w,*** such that each of ***m*** and ***w*** prefers the other to their partner in ***S.*** Does there always exist a perfect matching with no strong instability? Either give an example of a set of men and women with preference lists for which every perfect matching has a strong instability; or give an algorithm that is guaranteed to find a perfect matching with no strong instability.
2. A **weak instability** in a perfect matching ***S*** consists of a man ***m*** and a woman ***w,*** such that their partners in S are ***w’*** and ***m’***, respectively, and one of the following holds:
   1. ***m*** p refers w to ***w’***, and ***w*** either prefers m to ***m’*** or is indifferent between these two choices; or
   2. ***w*** prefers m to ***m’***, and ***m*** either prefers ***w*** to ***w’*** or is indifferent between these two choices. In other words, the pairing between ***m*** and ***w*** is either preferred by both, or preferred by one while the other is indifferent. Does there always exist a perfect matching with no weak instability? Either give an example of a set of men and women with preference lists for which every perfect matching has a weak instability; or give an algorithm that is guaranteed to find a perfect matching with no weak instability.